

I. PART 1. MULTIPLE CHOICES QUESTIONS (7.0 points)

Write the correct answer (A, B, C or D) for each of the following questions in the correspondingly numbered space on your answer sheet.

Question 1. Given trapezoid $ABCD$ with AB and CD parallel and $\hat{A} - \hat{D} = 40^\circ, \hat{B} = 5\hat{C}$. Then $\hat{A} + \hat{B}$ is equal to

- A. 100° B. 260° C. 140° D. 220°

Question 2. Let $ABCD$ be a parallelogram such that $\widehat{ADB} = 90^\circ$, $AB = 5, AD = 3$, then AC is

- A. 8 B. 4 C. $\sqrt{13}$ D. $2\sqrt{13}$

Question 3. Given the lengths of sides of a rectangular are $6cm$ and $8cm$, respectively. What is the length of the diagonal?

- A. $10cm$ B. $12cm$ C. $6cm$ D. $8cm$

Question 4. Let n be an integer. Suppose that $n^2 + n - 2$ is divisible by $n + 1$. What is the value of n ?

- A. 3 B. 2 C. 1 D. 4

Question 5. A square is inscribed inside a rhombus with diagonals 6 and 10. Find the area of the square.

- A. $\frac{225}{64}$ B. $\frac{225}{16}$ C. $\frac{15}{4}$ D. $\frac{15}{8}$

Question 6. Given square $ABCD$, E is mid-point of AB and F is point on BC such that $BF = 2FC$. If $AB = 6$, then EF is

- A. 6 B. 4 C. 5 D. 7

Question 7. Given rectangular $ABCD$ with $AB = 4, AD = 6$, M and N are two points on the sides AB and DC , respectively such that $AM = MB$ and $DN = 2NC$. The area of triangle DMN is equal to

- A. 4 B. 8 C. 12 D. 6

Question 8. Which of the following statements is false?

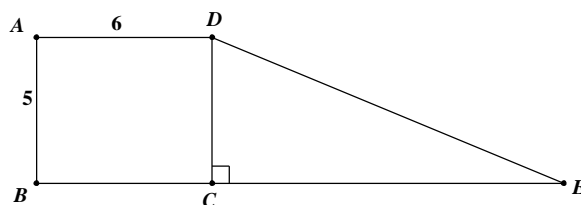
- A. $(a^2 - b^2)(a^2 + b^2) = a^4 - b^4$ B. $(2 - a)[(a + 1)^2 + 3] = 8 - a^3$
C. $(a^2 - b^2)(a + b) = a^3 - b^3$ D. $(b - a)(a^2 + b^2 + ab) = b^3 - a^3$

Question 9. Given expression $\frac{x+1}{x^2(x+2)} + \frac{3}{|x+2|}$. The expression is well-defined with which of the following conditions of x ?

- A. $x \neq 0$ B. $x \neq -2$ C. $x \neq 0$ and $x \neq -2$ D. $x \neq 0$ or $x \neq -2$

Question 10. On the right figure, given rectangle $ABCD$ and right triangle DCE which have the same area. The height DE is equal to

- A. $5\sqrt{2}$ B. $\sqrt{125}$
C. 13 D. 12



Question 11. Simplify the expression $\frac{1}{x^2-3x+2} - \frac{3}{4-2x}$?

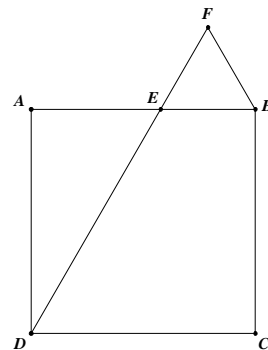
- A. $\frac{5-3x}{2(x^2-3x+2)}$ B. $\frac{3x-1}{x^2-3x+2}$ C. $\frac{1-3x}{2(x-1)(x-2)}$ D. $\frac{3x-1}{2(x-1)(x-2)}$

Question 12. Find the minimum value of the expression $2x^2 - 4|x-2| - 8x + 3$.

- A. -6 B. -7 C. -5 D. 3

Question 13. On the right figure, let $ABCD$ be a square, E and F are two points on AB and DE , respectively such that BEF is an equilateral triangle. If $AB = \sqrt{3}$, then EF is equal to

- A. $2 - \sqrt{3}$ B. $\frac{\sqrt{3}}{3}$
C. $\sqrt{3} - 1$ D. $\frac{\sqrt{3}}{2}$



Question 14. The value of expression $A = x^5 - 3x^2y^3 + 3x^3y^2 - y^5$ with $x = -2$ and $y = 3$ is equal to

- A. -383 B. -167 C. -815 D. -103

Question 15. Let $Q(x) = 2x^3 + ax^2 + bx + c$ where a, b, c are given constants. If $Q(0) = 1$, $Q(-1) = 2$, $Q(2) = 3$, the value of $a + b + c$ is equal to

- A. -2 B. $\frac{20}{3}$ C. 4 D. $-\frac{14}{3}$

Question 16. What is the value of x in $\frac{-3x^2 + 2x + 1}{x^2 + 3x - 4} = 1$?

- A. $x = -\frac{1}{3}$ B. $x = -\frac{5}{4}$
C. $x = 1$ or $x = -\frac{1}{3}$ D. Does not exist x

Question 17. Quadrilateral $ABCD$ is a rhombus with perimeter 48cm . The length of the side of rhombus is equal to

- A. 24cm B. 12cm C. 16cm D. 9cm

Question 18. Suppose that x and y are two integer numbers such that $x - y - xy = 79$ and $x^2 + y^2 = 130$.

The value of $(x - y + 1)^2$ is equal to

- A. 288 B. 289 C. 209 D. 260

Question 19. The quotient of the division of polynomial $2019x^{2018}y^{2020} - 3x^4y^6$ by polynomial $\frac{3}{2}x^3y^2$ is equal to

- A. $1346x^{2015}y^{2018} - 2xy^4$ B. $\frac{6057}{2}x^{2021}y^{2022} - \frac{9}{2}x^7y^8$
C. $1346x^{2021}y^{2022} - 2x^7y^8$ D. $\frac{6057}{2}x^{2015}y^{2018} - \frac{9}{2}xy^4$

Question 20. Factorize the polynomial $x^4 + 2020x^2 + 2019x + 2020$.

- A. $(x+1)(x^2+x+2020)$ B. $(x+2020)(x^3+2020x+2019)$
 C. $(x^2+x+1)(x^2-x+2020)$ D. $(x+1)^2(x^2-x+2020)$

Question 21. Calculate the sum of all coefficients in the expansion of expression $(x^2 - 2x - 3)^7$.

- A. -4 B. -78125 C. -16384 D. -2187

Question 22. Simplify the expression $\left(\frac{1-x^3}{1-x} - x\right) : \left(\frac{1-x}{1+x^2}\right)$?

- A. $\frac{x^2+x+1}{1+x^2}$ B. $1-x$ C. $\frac{(1+x^2)^2}{1-x}$ D. $\frac{1+x^2}{1-x}$

Question 23. The value of $A = \frac{x^3+27}{x+3} - \frac{x^4-16}{x^2+4}$ with $x = \frac{1}{4}$ is equal to

- A. $\frac{21}{5}$ B. $\frac{43}{4}$ C. $\frac{61}{4}$ D. $\frac{49}{4}$

Question 24. The coefficient of x^2 in the expansion of $(1+x)(2+x^2)(3+x^3)\dots(7+x^7)$ is equal to

- A. 1 B. 28 C. 5040 D. 2520

Question 25. Given trapezoid $ABCD$ with two bases $AB = 3cm$, $CD = 13cm$. Suppose that the area of the trapezoid is $48cm^2$. The height AH is equal to

- A. $3cm$ B. $24cm$ C. $1,5cm$ D. $6cm$

Question 26. Let a be a positive number such that $\left|a - \frac{1}{a}\right| = 1$. The value of $a^2 + \frac{1}{a^2}$ is equal to

- A. 1 B. -1 C. 3 D. 2

Question 27. The sum of all angles in a quadrilateral is equal to

- A. 180° B. 540° C. 270° D. 360°

Question 28. The common denominator of $\frac{1}{(x-1)(4x^2-4x+1)}$ and $\frac{x+1}{(2x-1)(x^2-2x+1)}$ is equal to

- A. $(x-1)^2(2x-1)$ B. $(x-1)^2(1-2x)^2$ C. $(2x-1)(x-1)$ D. $(2x-1)(4x^2-4x+1)$

Question 29. Suppose that polynomial $x^4 + 2x^3 + x^2 + m$ is divisible by polynomial $x^2 + x - 1$. The value of m is

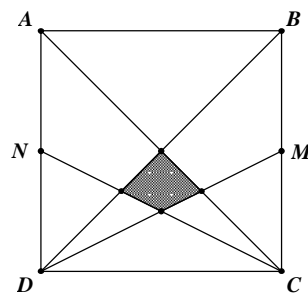
- A. 1 B. 4 C. -2 D. -1

Question 30. Given triangle ABC with the right angle A . Let M be a point on BC such that $BM = 2MC$. If $AB = 3cm$, $AC = 4cm$ then the area of triangle ACM is equal to

- A. $6cm^2$ B. $3cm^2$ C. $4cm^2$ D. $2cm^2$

Question 31. On the right figure, given square $ABCD$ with $AB = 10$. Assume that M and N are mid-points of the sides AD and BC , respectively. The area of the black space is equal to

- A. $\frac{25}{6}$ B. 5 C. $\frac{25}{12}$ D. $\frac{25}{7}$



Question 32. Let a, b be a real numbers such that $\frac{2x+1}{x^2+6x+9} = \frac{a}{x+3} + \frac{b}{(x+3)^2}$. The value of $a+b$ is equal to

- A. 7 B. -5 C. 1 D. -3

Question 33. Given trapezoid $ABCD$ with two bases AB, CD . Suppose that M and N are mid-points of the sides AD and BC . Let AC and BD intersect MN at I and K , respectively. If $AB = 6\text{cm}, CD = 8\text{cm}$ then IK is equal to

- A. 7cm B. 1cm C. 3cm D. 4cm

Question 34. What is the solution of $A = 2x^5 - \frac{1}{2}x^3 + \frac{3}{4}x^2 - 1$ with $x = -2$?

- A. 64 B. -58 C. -64 D. 58

Question 35. Let $ABCD$ be an isosceles trapezoid with two bases AB, CD and AC is perpendicular to BD . The line AC meets BD at K . If $AC = 12$ and $AK = 4$ then DC is equal to

- A. 4 B. $4\sqrt{2}$ C. 8 D. $8\sqrt{2}$

II. PART II. PROBLEM SOLVING (3.0 points)

Write the solutions to the following problems in the provided space on your answer sheet.

Problem 1. Find all prime numbers p such that $16p+1$ is a perfect cube.

Problem 2. A pair of numbers are *twin primes* if they differ by two and both are primes. Prove that, except the pair $\{3;5\}$, the sum of any pair of twin primes is a multiple of 12.

Problem 3. In rectangle $ABCD$, the length of side AB is twice as the length of side BC . A point P is taken on side AB such that $BP = \frac{1}{4}AB$. Show that BD is perpendicular to CP .

-THE END-

Student's full name: Student's ID:

First observer's name and signature: Second observer's name and signature: